

Homework 5

1. Determine the open intervals on which the graph of the function is concave upward or concave downward. (Enter your answers using interval notation. If an answer does not exist, enter DNE.)

$$f(x) = x^2 - 2x + 8$$

2. Find each limit, if it exists. (If an answer does not exist, enter DNE.)

(a) $\lim_{x \rightarrow \infty} \frac{4-7x}{4x^3-9}$

(b) $\lim_{x \rightarrow \infty} \frac{4-7x}{4x^{-9}}$

(c) $\lim_{x \rightarrow \infty} \frac{4-7x^2}{4x^{-9}}$

3. Analyze and sketch a graph of the function (Label any intercepts, relative extrema, points of inflection, and asymptotes)

$$y = \frac{2x}{9-x^2}$$

Sol :

1.

concave upward : $(-\infty, \infty)$

concave downward : *DNE*

2.

(a) 0

(b) $-\frac{7}{4}$

(c) *DNE*

3.

$$y = \frac{2x}{9 - x^2}$$

$$y' = \frac{2(x^2+9)}{(x^2-9)^2}, \text{ undefined when } x = \pm 3$$

$$y'' = \frac{4x(x^2+27)}{(x^2-9)^3} = 0 \text{ when } x = 0, \text{ undefined when } x = \pm 3$$

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x \pm 3$

	y	y'	y''	Conclusion
$-\infty < x < -3$		+	+	Increasing, concave up
$-3 < x < 0$		+	-	Increasing, concave down
$x = 0$	0	+	0	Point of inflection
$0 < x < 3$		+	+	Increasing, concave up
$3 < x < \infty$		+	-	Increasing, concave down

